Multi-Impulse to Time Optimal Finite Burn Trajectory Conversion

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Abstract (condensed)

A novel conversion algorithm is presented that combines the fidelity of indirect optimization methods with the generality of direct methods to more easily solve time-optimal, finite-burn pseudo-rendezvous problems. An algorithm is described that converts a set of multiple-impulses, representing high- or low-thrust maneuvers, to an exact time optimal finite-burn trajectory for a thrust limited, constant exhaust velocity spacecraft. A pseudo-rendezvous problem yields a solution whose final time, position and velocity state is equal to that of the original post-impulsive trajectory. An iterative adjoint-control transformation and sequential constraint vector are used to solve the optimal control two-point boundary value problem. Examples are shown for both high and low-thrust non-coplanar Earth orbit transfers, as well as a low-thrust Hohmann-type Earth-Mars transfer.

Abstract (extended)

A novel conversion algorithm is presented which extends a hybrid optimization method [1] to a new problem domain. When attempting to solve general optimal control finite-burn trajectory problems (e.g. low-thrust orbit transfers), difficulty arises in autonomously deriving a sufficient initial costate vector at ignition to enable solution convergence. Due to the extreme sensitivity of solution convergence to these initial costates, optimal control methods are often difficult to generalize and automate, as prior knowledge of the problem domain is required [2-7]. Despite these difficulties, solutions to the finite-burn optimal control problem are highly desired, as they represent the highest fidelity solutions available, often associated with a savings in cost (propellant, burn time, transfer time) over direct parameter optimization methods. The presented method mitigates the difficulty in generating the approximation of the initial costates by iteratively applying adjoint control transformations and slowly introducing the full constraint vector with each iterate. This provides an increasingly more accurate approximation of the initial costates, which by the end of the process become sufficiently close to the optimal values, enabling solution convergence. This method requires no prior knowledge of the solution, which adds simplicity to the design process and aids automation.

The presented approach combines the fidelity of indirect methods with the robustness of direct methods to more easily solve general time-optimal, finite-burn problems. An algorithm is herein described that converts a set of multiple-impulses, representing the entirety or a portion of a high- or low-thrust maneuver, to an exact time optimal finite-burn trajectory for a thrust limited, constant exhaust velocity spacecraft. This can be called a pseudo-rendezvous problem, as it yields a finite-burn solution whose final time, position and velocity state is equal to that of the original post-multi-impulse trajectory [Fig. 1]. An extended adjoint-control transformation is used to initialize the optimal control Two-Point Boundary Value Problem (TPBVP). The power of this approach is best seen when applied to low thrust transfers, but is equally applicable to high thrust problems, with less savings in objective cost compared to direct solutions.

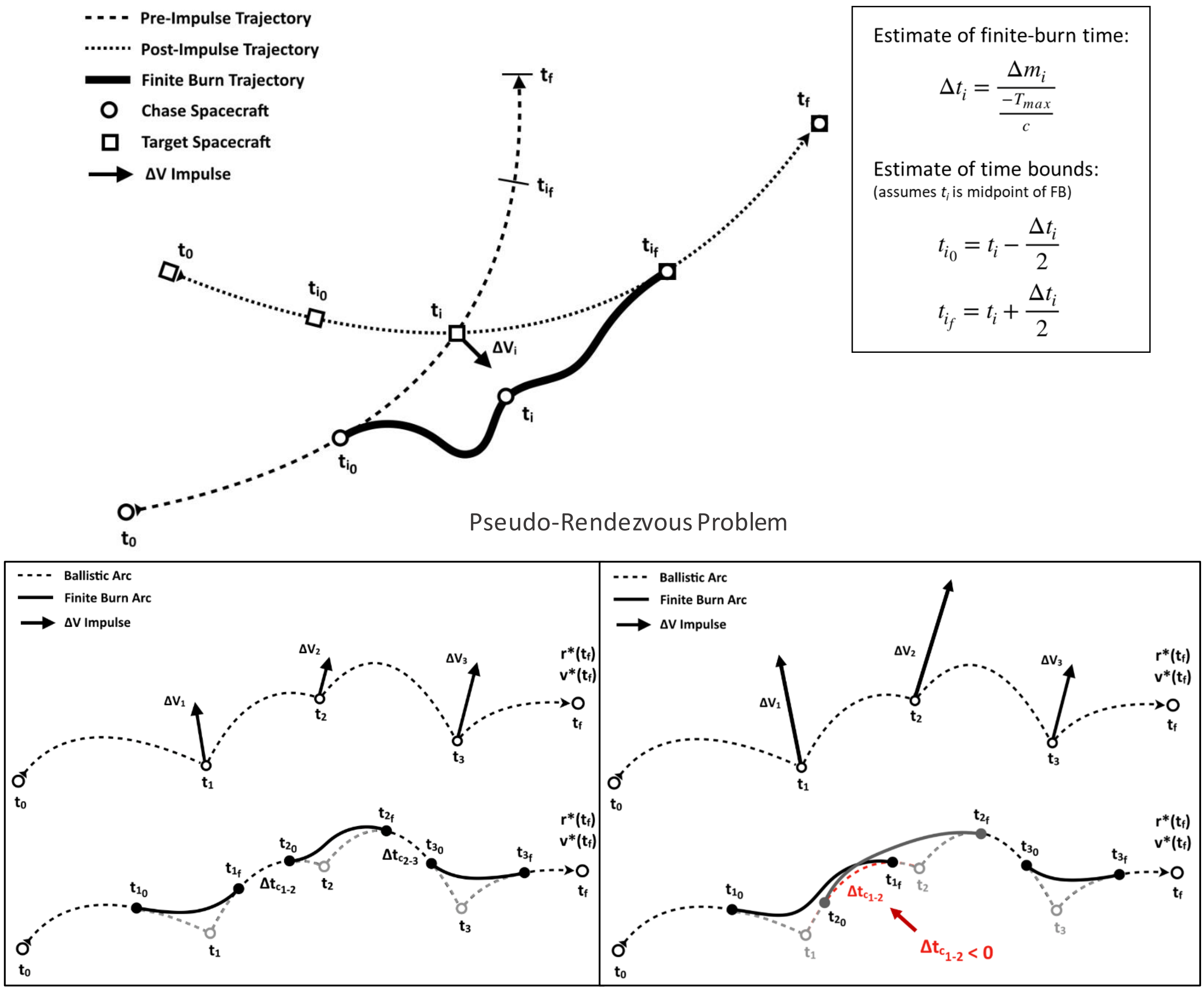

Figure 1. [Top] Single Impulse to Finite Burn Conversion [1]**,** with initial estimates of time bounds based on the impulse. [Bottom-Left] Multiple-Impulse to Finite Burn Conversion, with interior coast segments between finite burns. [Bottom-Right] Finite burn overlap problem (unmitigated), with negative interior coast between finite-burns.

The adjoint-control transformation central to the solution method represents a conversion between three fundamental burn models: Impulsive, Sub-Optimal Control (SOC), and Optimal Control Theory (OCT) [Fig. 2]. The impulsive model is comprised of ΔV magnitudes and angular directions, with instantaneous mass change. The SOC model represents a sub-optimal finite-burn, with thrust directions as a function of polynomial-based steering angles. The OCT model represents an optimal control finite-burn, with state and costate dynamics derived via the standard formulation of the optimal control TPBVP [1]. Selecting the proper parametrizations within these models allows for the transfer of solution information from one model to another, for instance using a converged impulsive solution to seed an OCT solution. This is critical, as the three burn models increase in convergence difficulty for general

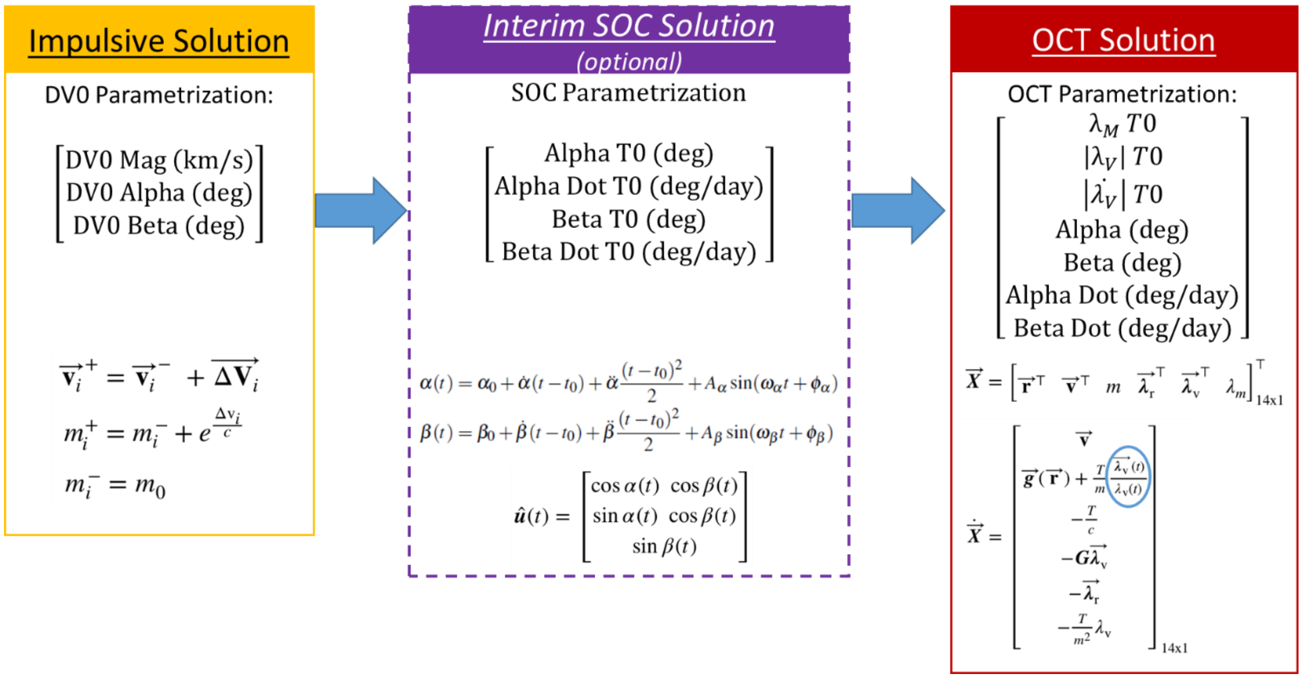

Figure 2. Conversion algorithm burn model parametrizations. DV0 represents the ΔV impulse (magnitude and direction), while Alpha & Beta angles represent thrust or impulse direction (as defined in any specified frame, e.g. inertial J2000, 2-body rotating, VUW). Note the primer vector in the OCT dynamics, which controls thrust direction.

optimization problems. This is why starting from an impulsive solution and incrementally converting to higher fidelity models (e.g. OCT) is a favorable strategy for some complex finite-burn mission design problems (e.g. low-thrust).

The presented multi-impulse to finite-burn conversion procedure [Fig. 4] is based on the single-impulse conversion algorithm by Jesick and Ocampo [1], with optimization set up in [Fig. 3]. A sequence of impulses is converted into time-optimal finite-burns one by one. Focus is then given to the finite-burn arc overlap problem, which can arise when patching together a series of single-impulse conversions with large ΔVs relative to the engine performance [Fig. 1]. An overlap is said to exist where the coast time between two sequential finite-burn segments is negative. The overlap problem is mitigated through a process of introducing additional boundary constraints, and by relaxing the pseudo-rendezvous state targeting on all interior conversions, leaving the state targeting for only the final impulse conversion. This redefinition of the optimal control problem allows the interior finite-burns to be re-optimized, while ensuring the final pseudo-rendezvous targeting is upheld, which is the stated goal of the procedure.

The conversion algorithm is modeled in NASA JSC's in-house orbit trajectory optimization software Copernicus [8], adding new capability to its suite of mission design tools. Numerical examples are shown for both high and low-thrust non-coplanar Earth orbit transfers, as well as a low-thrust Hohman-type Earth-Mars transfer. Preliminary conversion results are shown in [Fig. 5-6], with comparisons to conventionally computed, sub-optimal control (SOC) solutions. The converted OCT solutions are shown to offer improved accuracy, and in all cases cheaper propellant cost, when compared to sub-optimal SOC solutions.

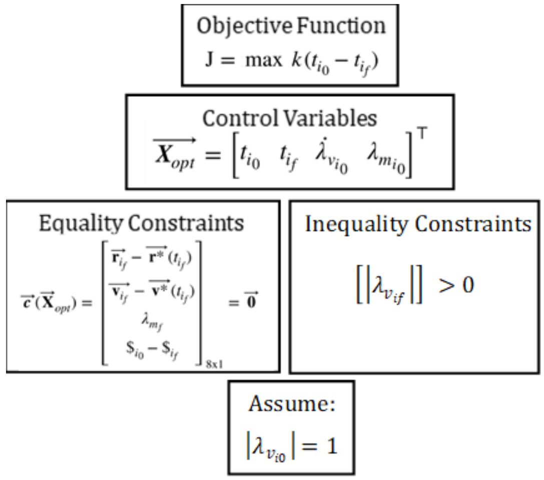

Figure 3. Optimization solution strategy derived by Jesick & Ocampo for single impulse to finite-burn conversion, forward shooting, utilizing pseudo-rendezvous targeting [1].

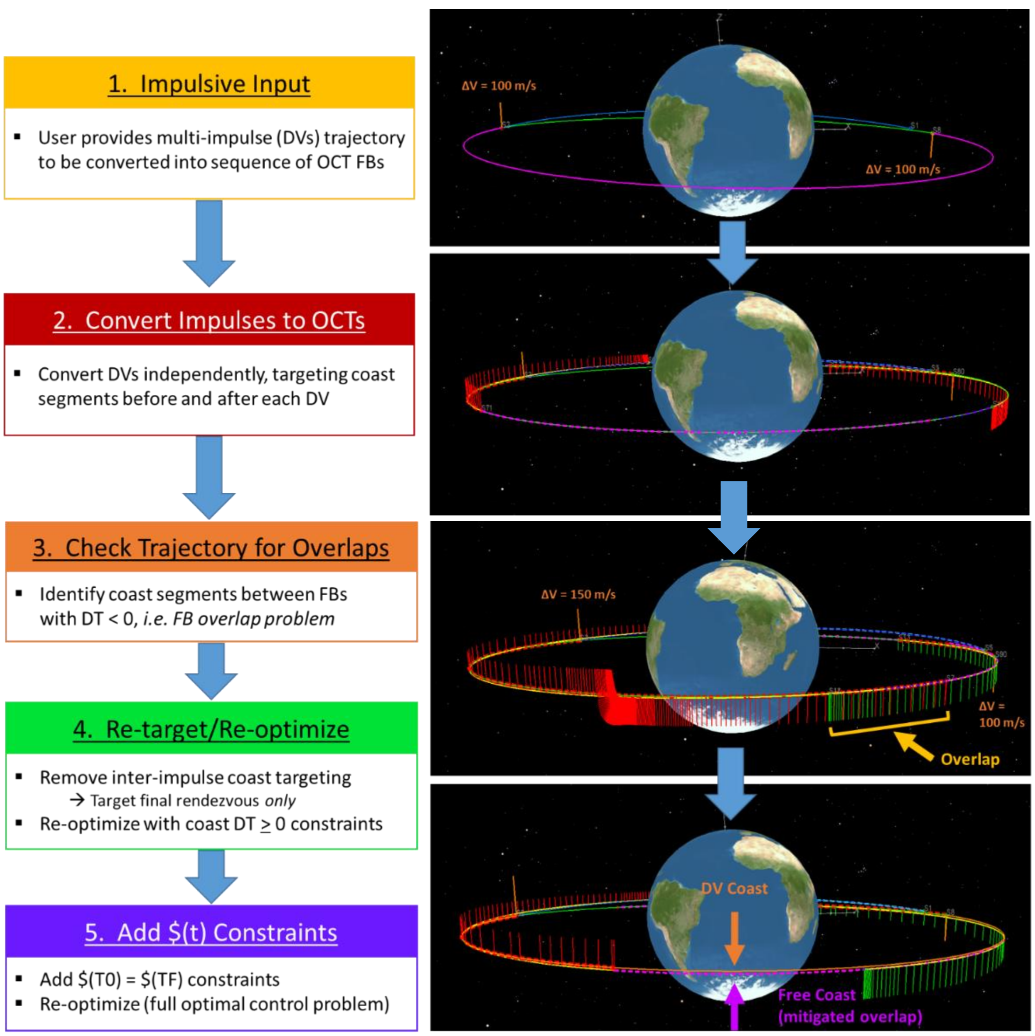

Figure 4. Finite-burn (FB) conversion algorithm, with example low-thrust inclination change problem. Note the addition of the final equality constraint, the switching function $(t), in the final step to avoid over constraining the earlier problem steps.

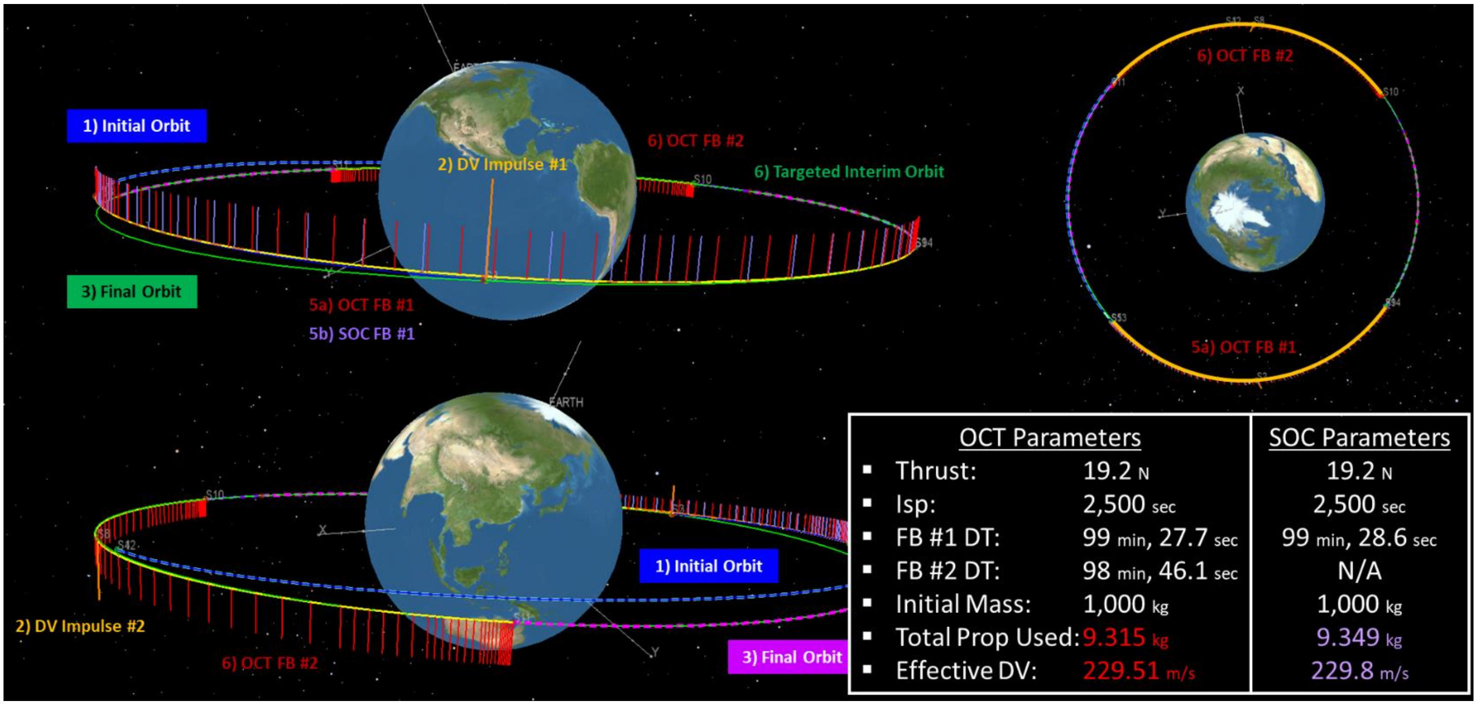

Figure 5. Two-impulse inclination change, converted to low-thrust OCT finite-burn, with comparisons to SOC solution.
Δi = 1.146 deg / impulse
ΔV = 100 m/s / impulse

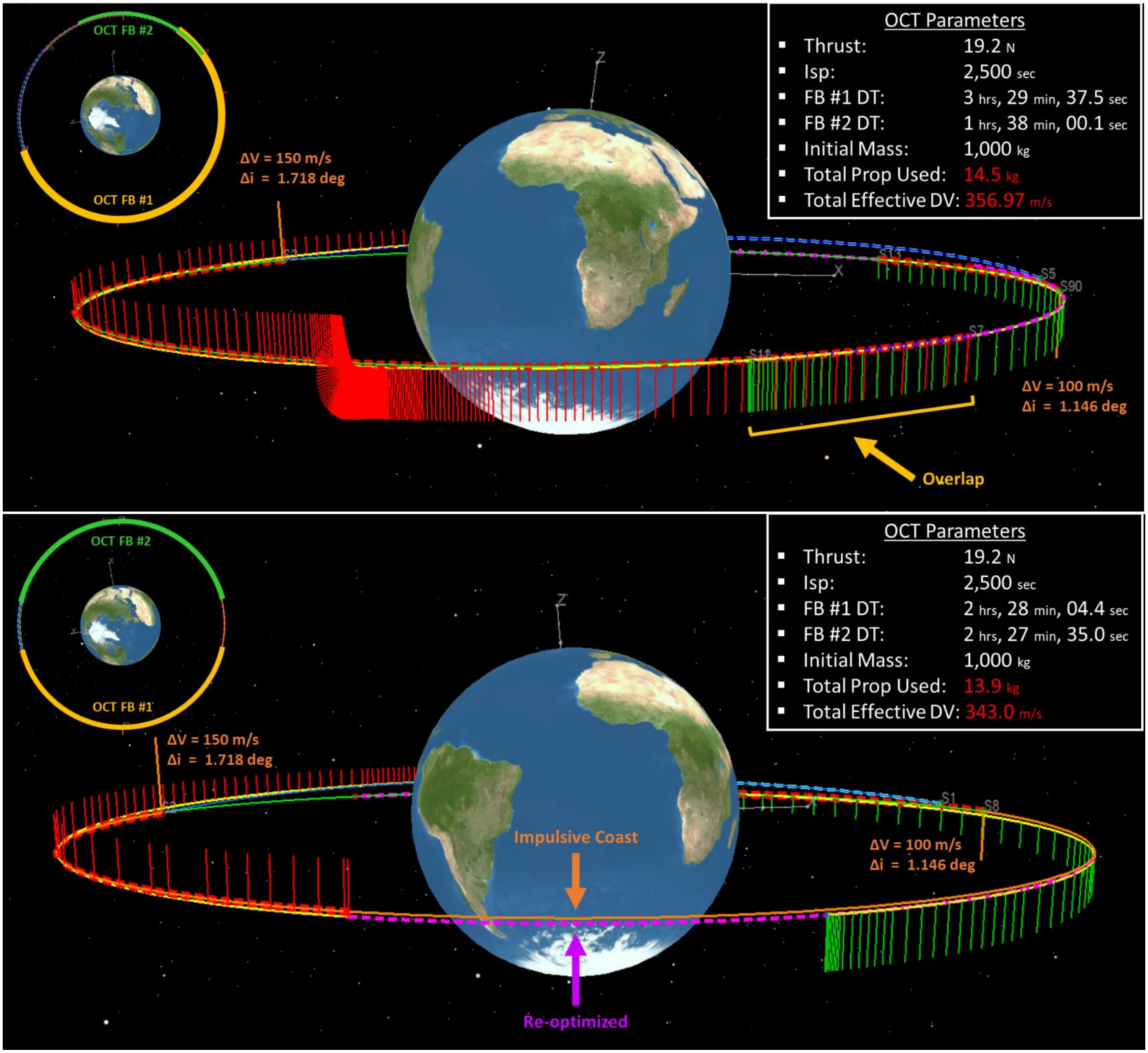

Figure 6. Finite-burn arc overlap problem, unmitigated (top) vs. mitigated (bottom): two-impulse inclination change, converted to low-thrust finite-burn. Overlap caused by increased ΔV magnitude of first impulse (requiring a longer corresponding finite-burn for stated engine parametrization).

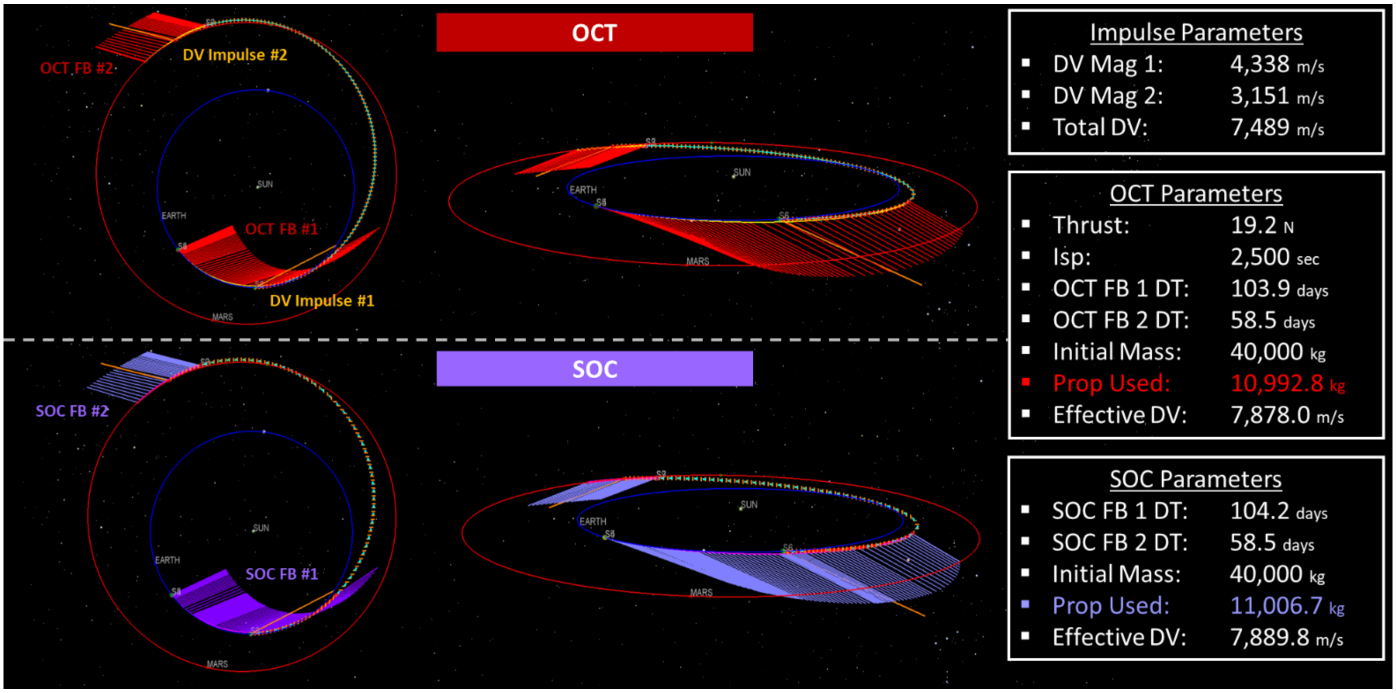

Figure 7. Earth-Mars, non-coplanar two-impulse Hohmann Transfer to low-thrust conversion, with planetary departure and arrival $C_3 = 0$.

References

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